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Prediction of Traffic Flow Time Series Data on Jakarta-Cikampek Toll Road Using a Chaotic Approach and Local Linear Approximation Method

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ABSTRACT

The Jakarta–Cikampek Toll Road is widely recognised as a critical corridor linking Jakarta with major industrial centres and expanding residential areas in West Java. Traffic along this route is frequently dense and exhibits noticeable fluctuations over time. At certain periods, these variations reveal nonlinear patterns shaped not only by commuter movements and freight transport but also by broader economic activity. This study focuses on short-term traffic forecasting by modelling traffic flow as a nonlinear dynamical system rather than a purely stochastic process. A chaos-based framework is applied to the traffic flow time series through two main stages: first, the presence of chaotic behaviour is examined using the 0–1 test; second, short-term forecasting is performed using the Local Linear Approximation Method (LLAM), which utilises neighbouring trajectories in phase space. Hourly traffic flow data recorded over seven consecutive days are analysed, with 144 observations used for training and 24 for testing. The 0–1 test confirms the existence of chaotic dynamics within the series. During the forecasting stage, LLAM demonstrates reasonable agreement with observed traffic flow, achieving a Pearson correlation coefficient (r) of 0.8964, a Mean Absolute Error (MAE) of 159.41 vehicles per hour, and a Root Mean Squared Error (RMSE) of 203.42 vehicles per hour. These findings indicate that chaos-based modelling provides a dependable approach for short-term traffic forecasting. Beyond predictive accuracy, the framework offers practical value by supporting quicker operational responses, improved congestion management, and more effective short-term planning in dynamic toll-road environments.

Keywords: Traffic Flow Forecasting; Chaos Approach; Local Linear Approximation Method; Time Series Analysis; Jakarta–Cikampek Toll Road.

Nomenclature

τ	time delay used in phase-space reconstruction, determined via average mutual information (hours)
d	embedding dimension, indicating the number of coordinates in each state vector (dimensionless)
r	Pearson correlation coefficient between observed and predicted values (dimensionless, range -1 to 1)
T	forecasting horizon, representing the number of steps ahead (hours)
k	number of nearest neighbours used in LLAM prediction (count)
$E1(d)$, $E2(d)$	Cao method parameter for identifying optimal embedding dimension (dimensionless)
$X(j)_{train}$	training traffic flow time series used during the training phase (vehicle/hour)
$X(j)_{test}$	testing traffic flow time series used during the testing phase (vehicle/hour)

Abbreviations

LLAM	local linear approximation method, forecasting technique exploiting local linearity in phase-space trajectories
CCTV	closed circuit television, monitoring units installed along the Jakarta–Cikampek Toll Road to record hourly traffic flow

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AMI	average mutual information, applied to determine the optimal time delay in time series analysis
MAE	mean absolute error, average magnitude of prediction errors (vehicles/hour)
RMSE	root mean square error, an accuracy metric emphasizing larger deviations (vehicles/hour)

1.0 INTRODUCTION

Traffic congestion continues to pose a major challenge in large urban areas, particularly along road corridors that serve strategic economic functions [1]. The Jakarta–Cikampek Toll Road is one of Indonesia's most critical transport links, connecting the capital city with key industrial regions such as Bekasi, Karawang, and Purwakarta [2], [3]. Traffic conditions along this corridor are often difficult to predict, especially during homecoming and holiday seasons when sharp increases in vehicle volume occur [4], [5]. Rising vehicle ownership and diverse driving behaviours further intensify these fluctuations, directly affecting toll-road operations and potentially influencing regional economic activity.

From a modelling perspective, traffic flow represents a highly complex system. Demand, capacity, and temporal patterns rarely follow simple linear relationships [6]. Even small disturbances in traffic conditions can lead to significant changes in vehicle density and flow characteristics. Irregular fluctuations often appear unexpectedly, complicating short-term forecasting [7]–[9]. Similar to hydrological systems, traffic flow exhibits nonlinear behaviour and sensitivity to initial conditions [10], [11]. Chaos theory shows that such deterministic systems remain highly sensitive to their initial states, limiting reliable prediction to relatively short time horizons [12].

Hydrological studies demonstrate that recognising chaotic properties before forecasting can improve the accuracy of predictions for highly variable time series, such as water levels [13]. Approaches grounded in chaos aim to uncover deterministic structures through phase-space reconstruction and dynamical system analysis [14]. Confirming the presence of chaos is therefore an essential step in choosing appropriate forecasting strategies and in defining the practical limits of prediction [15]. Once chaotic behaviour is confirmed, local forecasting methods such as the Local Linear Approximation Method (LLAM) can be applied by analysing neighbouring trajectories in phase space [16]. LLAM has been successfully used in applications ranging from ozone prediction [17] and urban traffic forecasting [18] to reservoir water level estimation [13] and carbon monoxide concentration analysis [19], demonstrating competitive predictive performance.

Although interest in nonlinear traffic modelling has increased [6]–[9], [14], explicit validation of chaotic dynamics in Indonesian toll-road systems remains limited, particularly in the context of short-term forecasting. Without confirming the underlying dynamical structure, forecasting models risk failing to capture the deterministic nonlinear behaviour embedded in traffic flow data [21], [22]. For this reason, systematic identification of chaos, followed by the application of suitable local forecasting methods, is essential in managing highly dynamic toll-road environments.

This study applies a chaos-based methodology in combination with LLAM to analyse traffic flow data from the Jakarta–Cikampek Toll Road. The main contribution lies in developing and validating a chaos-informed LLAM framework for short-term forecasting in this dynamic setting. Unlike earlier studies that applied forecasting techniques without first verifying the underlying dynamical structure, this research incorporates explicit chaos validation using the 0–1 test before implementing LLAM-based forecasting. In addition, it provides empirical evidence of chaotic dynamics within an Indonesian toll-road system, a context that has received relatively little attention in the existing traffic forecasting literature. The findings highlight the presence of deterministic nonlinear characteristics in the traffic system and offer practical insights to support operational traffic management.

2.0 METHODOLOGY

In this research, the analytical approach inspired by chaos theory was employed in the forecasting of the flow of traffic in the near future. To ensure that the forecasting method proposed in this research is practical, yet taking into consideration that the system being analysed is nonlinear, the proposed method was broken down into four simple steps. The four steps include describing the data, identifying chaotic patterns, reconstructing phase space, and finally, using the Local Linear Approximation Method (LLAM) to create short-term predictions.

2.1 Data description

The data set shows how traffic moves on the Jakarta-Cikampek Toll Road in West Java, Indonesia. The data were collected throughout a week, from April 7 to April 13, 2025, by PT Jasa Marga. CCTV cameras were used to record the data. There were 168 data points in all, and each one shows how many cars crossed the toll road in an hour. The data were split into two groups for modeling: one for training and one for testing. There were 144 data points for training and 24 data points for testing. The data points together made up a time series.

The traffic flow time series is expressed as a single observed sequence:

$$X(j) = \{x_1, x_2, x_3, ..., x_{168}\},\ j = 1, 2, ..., N \quad \text{(vehicles/hour)} \qquad (1)$$

Accordingly, the training and testing datasets are defined as:

$$X(j)_{train} = \{x_1, x_2, x_3, ..., x_{144}\} \quad \text{(vehicles/hour)} \qquad (2)$$

$$X(j)_{test} = \{x_1, x_2, x_3, ..., x_{24}\} \quad \text{(vehicles/hour)} \qquad (3)$$

A first look at the dataset shows strong daily trends, with clear peaks and troughs that are related to commuting activity, as well as shorter-term changes that are not as regular. Figure 1 shows the hourly traffic flow time series and the difference between the training and testing periods. It also shows clear peaks, troughs, and short-term changes that make it necessary to utilise chaos-based analytical methods in the next steps.

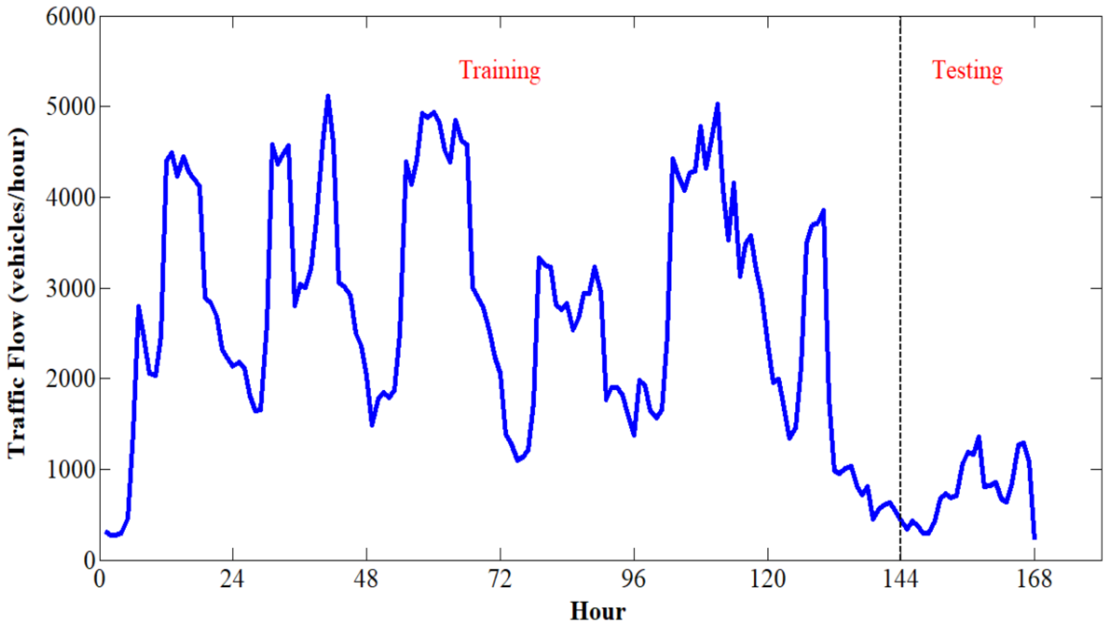

Figure 1. Time series of hourly traffic flow over seven days, split into training (Hours 1–144) and testing (Hours 145–168) periods

2.2 Chaos detection

Prior to forecasting, it is essential to verify whether the traffic flow time series exhibits chaotic behaviour. Thisverification ensures that the subsequent nonlinear modelling approach is theoretically justified. In this study, the 0–1 test for chaos is employed due to its simplicity and effectiveness in distinguishing chaotic dynamics from regular or stochastic behaviour [20], [21]. Let $X(j)$, with $j = 1, 2, \ldots, N$, denote the traffic flow training data, where $N = 144$. Two transformed variables are defined as:

$$p_c(n) = \sum_{j=1}^{n} X(j)\cos(jc) \qquad (4)$$

$$q_c(n) = \sum_{j=1}^{n} X(j)\sin(jc) \qquad (5)$$

where $c \in (0, \pi)$ is a randomly selected constant. Based on these transformations, the mean square displacement is computed as:

$$M_c(n) = \lim_{N\to\infty} \frac{1}{N}\left(\left[p_c(j+n)-p_c(j)\right]^2+\left[q_c(j+n)-q_c(j)\right]^2\right), \tag{6}$$

with $n \leq N$. Following Gottwald and Melbourne [22], a cutoff value is introduced such that:

$$n_{cut} = \frac{N}{10} \tag{7}$$

For the training dataset ($N = 144$), the cutoff value becomes $n_{cut} = 14.4$ which is rounded down to 14 to maintain compatibility with discrete-time analysis. The asymptotic growth rate is then estimated as:

$$K_c = \lim_{n\to\infty} \frac{\log M_c(n)}{\log n}, \tag{8}$$

The procedure is repeated for several randomly chosen values of c, and the median value of K_c is used. According to [23], values of K_c close to 1 indicates chaotic dynamics, while values close to 0 suggest non-chaotic behaviour. Once chaotic behaviour is confirmed, the system dynamics are reconstructed in a multidimensional phase space.

2.3 Phase-space reconstruction

After confirming chaotic dynamics, phase-space reconstruction is performed to recover the underlying system dynamics from the one-dimensional time series. The reconstructed state vectors are defined as:

$$Y_j = \left\{x_j, x_{j+\tau}, x_{j+2\tau}, \ldots, x_{j+(d-1)\tau}\right\}, \tag{9}$$

where τ denotes the time delay, and d is the embedding dimension. The optimal time delay is determined using the Average Mutual Information (AMI) method [23], defined as:

$$I(T) = \frac{1}{N}\sum_{\alpha=1}^{N} p\left(u_\alpha, u_{\alpha+T}\right)\log_2\left(\frac{p\left(u_\alpha, u_{\alpha+T}\right)}{p(u_\alpha)p(u_{\alpha+T})}\right) \tag{10}$$

where $p(u_\alpha)$ and $p(u_{\alpha+T})$ represent the marginal probabilities, and $p(u_\alpha, u_{\alpha+T})$ denotes the joint probability. The selected time delay τ corresponds to the first minimum of $I(\tau)$.

The embedding dimension d is determined using Cao's method [24], expressed as:

$$E1\left(d\right) = \frac{E\left(d+1\right)}{E(d)} \tag{11}$$

with,

$$E\left(d\right) = \frac{1}{N - d\tau}\sum_{n=1}^{N-d\tau} \frac{\left\|Y_n^{d+1} - Y_{jj}^{d+1}\right\|}{\left\|Y_n^{d} - Y_{jj}^{d}\right\|} \tag{12}$$

where $\|\bullet\|$ denotes the Euclidean norm and Y_{jj}^d represents the nearest neighbour of Y_j^d. The optimal embedding dimension d is selected when $E1(d)$ stabilises within the interval $0.9 \leq E1(d) \leq 1.0$ [25].

2.4 Local Linear Approximation Method (LLAM)

Following phase-space reconstruction, short-term forecasting is performed using the Local Linear Approximation Method (LLAM). LLAM exploits the locally linear behaviour of neighbouring trajectories in the reconstructed phase space by approximating the system evolution through local linear mappings, enabling one-step-ahead prediction of chaotic time series. For each state vector, the future state is expressed as:

$$Y_{j+T}^{d} = AY_{j}^{d} + B, \quad (13)$$

where T represents the forecasting horizon, the parameters A and B are estimated using the least squares method based on the k nearest neighbours [24], [25]:

$$k = 2d, \quad (14)$$

Following the standard practice in local phase-space forecasting, the number of nearest neighbours is selected as $k = 2d$, which ensures sufficient local information for stable parameter estimation.

Forecasting performance is evaluated using the Pearson correlation coefficient (r), Mean Absolute Error (MAE), and Root Mean Square Error (RMSE), defined as:

$$r = \frac{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)\left(\hat{y}_i - \bar{\hat{y}}\right)}{\sqrt{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2\left(\hat{y}_i - \bar{\hat{y}}\right)^2}} \qquad r \in \left[-1,1\right] \quad (15)$$

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right| \qquad \text{(vehicles/hour)} \quad (16)$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2} \qquad \text{(vehicles/hour)} \quad (17)$$

where y_i and $\hat{y}_i$ denote the observed and predicted traffic flow values, respectively, and n is the number of testing samples.

3.0 RESULTS AND DISCUSSION

The presence of chaotic dynamics in the hourly traffic flow time series of the Jakarta–Cikampek Toll Road was examined using the 0–1 test for chaos. The cut-off parameter was set to $n_{cut} = \frac{N}{10} \approx 14$, corresponding to the length of the training dataset (N = 144), following the recommendation of Gottwald and Melbourne [22], where the cut-off value is commonly set to one-tenth of the data length. The parameter c was randomly selected from the interval $\left(0, \pi\right)$, and the median value of the growth rate K was used to ensure robustness. As summarised in Table 1, the median growth rate $K \approx 0.91$ satisfies the criterion $0.05 < K < 1$, indicating deterministic chaotic dynamics.

Table 1: Result of the 0–1 test for chaos

Parameter	Value K
0–1 test	0.9132

3.1 Phase Space Reconstruction Parameters

The optimal time delay τ was determined using the Average Mutual Information (AMI) method, as defined in Equation (10), and applied to the training dataset. The AMI values for different delays are reported in Table 2. Following the standard criterion of selecting the first local minimum of the AMI function, which indicates minimal redundancy between delayed components, the first local minimum was observed at $\tau = 2$. Accordingly, $\tau = 2$ was selected for phase space reconstruction. It should be noted that the optimal delay τ depends on the characteristics of the dataset and sampling resolution; therefore, different values of τ may be obtained across studies. The AMI values were computed directly from the training dataset using MATLAB R2009a, following a standard histogram-based estimation.

Table 2: Average Mutual Information (AMI) values for different time delays

T	1	2	3	4	5	6	7	8	9	10
I(T)	2.412	1.963	2.031	2.112	2.183	2.244	2.289	2.331	2.364	2.392

As shown in Table 2, the AMI decreases from $T = 1$ and reaches its first local minimum at $T = 2$. Therefore $\tau = 2$ is selected for the subsequent analysis. Following the standard criterion, this value is adopted as the optimal time delay for phase space reconstruction. To determine the appropriate embedding dimension, the Cao method was applied to the traffic flow time series, and the resulting $E1(d)$ and $E2(d)$ statistics are summarised in Table 3.

Table 3: Results of Cao's method for determining the embedding dimension

d	1	2	3	4	5	6	7	8	9	10
E1(d)	0.423	0.684	0.842	0.906	0.908	0.910	0.912	0.914	0.915	0.916
E2(d)	0.882	0.861	0.835	0.812	0.808	0.806	0.804	0.803	0.802	0.801

As reported in Table 3, the value of $E1(d)$ increases with the embedding dimension and begins to stabilise at $d = 4$. Meanwhile, $E2(d)$ deviates from unity for all examined dimensions, indicating that the traffic flow time series exhibits deterministic nonlinear dynamics rather than stochastic behaviour. While $E1(d)$ is used as the primary criterion for selecting the embedding dimension, $E2(d)$ is reported to provide additional confirmation of the deterministic nature of the traffic flow time series, following Cao (1997). Accordingly, the optimal embedding dimension is selected as $d = 4$, which is consistent with previous applications of the Cao method to traffic flow and other complex dynamical systems.

3.2 Short-term traffic flow forecasting using improved LLAM

A short-term traffic flow forecast was performed using the reconstructed phase space with time delay $\tau = 2$ and embedding dimension $d = 4$. The Local Linear Approximation Method (LLAM) was used to make the one-step-ahead forecast. To reduce noise, the vectors in the phase space were averaged across the columns. This improved numerical stability. The training set had 144 data points, and the testing set included 24.

Figure 2 demonstrates that the LLAM-based forecast generally follows the observed traffic pattern, although minor deviations are present. The two sets of data go up and down in comparable ways. The model captures short-term traffic variations with only minor deviations attributable to inherent data variability. The strong Pearson correlation (r) of 0.8964 supports this. The Mean Absolute Error (MAE) for the 24-sample testing dataset is 159.41 vehicles per hour, and the Root Mean Squared Error (RMSE) is 203.42 vehicles per hour. These figures show that the system can reliably measure how traffic moves in the short term, even as it changes.

A closer examination indicates that prediction errors tend to occur more frequently during rush hours. These discrepancies are difficult to eliminate, especially in systems that are driven by chaotic dynamics, because they depend on small changes in the starting conditions and short-term demand fluctuations. This is consistent with short-term projections, given the rapid pace of change. The dataset analysed in this study consists of one week of hourly traffic observations, chosen to reflect the primary objective of short-term forecasting under dynamic traffic conditions. Chaos-based local forecasting methods primarily focus on identifying short-term deterministic structures, which can be effectively represented within high-resolution but limited time windows. When applied to longer datasets that capture seasonal or structural variations, however, model performance may change. Extending the observation period in future research would allow for a more comprehensive evaluation of the method's robustness and its capacity to generalise across longer time horizons.

Conventional statistical models such as ARIMA and data-driven approaches like LSTM have been widely used in traffic flow forecasting [6]–[9]. These methods often deliver strong predictive performance, particularly when large datasets are available for parameter training and optimisation. In contrast, the chaos-based LLAM framework places emphasis on identifying deterministic nonlinear structures through phase-space reconstruction, offering a lighter alternative for short-term forecasting under dynamic traffic conditions. Rather than serving as a replacement for machine-learning approaches, this method provides a complementary perspective grounded in nonlinear dynamical system analysis.

Although the empirical validation in this study focuses on the Jakarta–Cikampek Toll Road, the proposed modelling framework is not confined to a single location. The procedure, beginning with chaos identification using the 0–1 test and followed by local forecasting through LLAM, can be applied to other toll-road systems that display nonlinear and dynamically evolving traffic patterns. Nevertheless, parameters such as embedding dimensions, delay selection, and neighbourhood structures must be recalibrated to reflect the specific characteristics of each dataset. Thus, while the methodological framework is transferable, its operational configuration should be carefully adapted to local traffic dynamics before implementation.

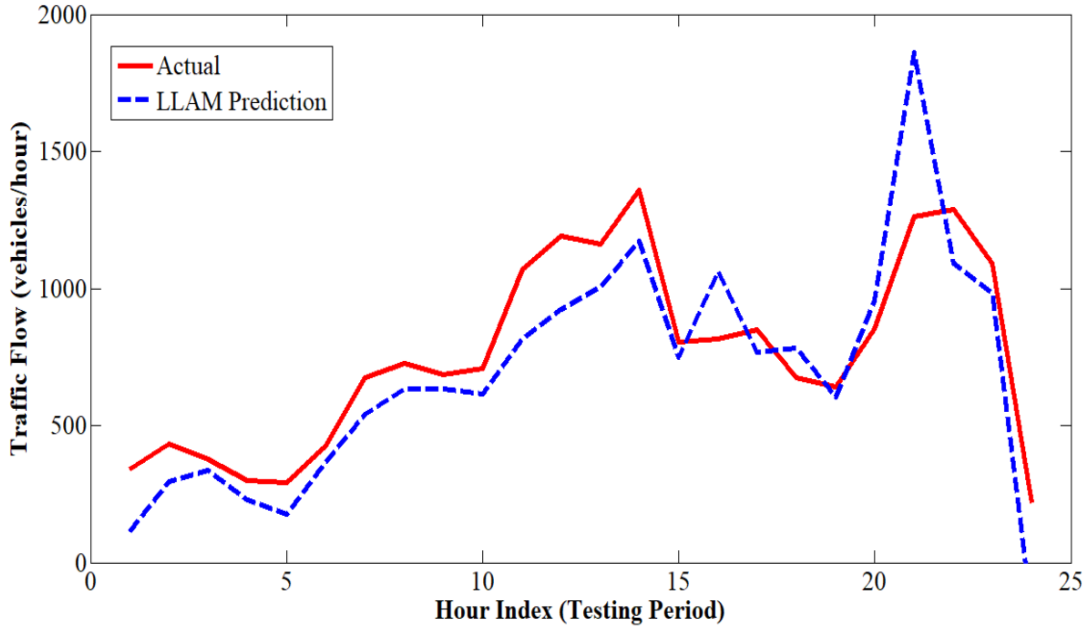

Figure 2. Traffic flow and LLAM one-step-ahead forecasts were measured during the testing period ($N = 24$)

4.0 CONCLUSION

This study shows that combining chaos theory with the Local Linear Approximation Method (LLAM) offers a practical framework for short-term traffic flow forecasting on the Jakarta–Cikampek Toll Road. By modelling traffic as a nonlinear dynamical system and validating the presence of chaotic characteristics before forecasting, the approach is able to capture deterministic structures embedded in highly dynamic traffic conditions. The findings suggest that chaos-informed forecasting can strengthen short-term prediction in environments characterised by nonlinear fluctuations, while also providing operational value for toll-road managers in anticipating congestion, optimising lane use, adjusting traffic control measures, and improving real-time decision-making during peak periods. The relatively simple computational requirements of LLAM further support its integration into existing traffic monitoring systems. Nonetheless, as with other short-term forecasting models, predictive performance remains sensitive to data variability and rapid changes in traffic dynamics. Future work could explore combining chaos-based modelling with machine learning techniques to improve robustness and forecasting accuracy.

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AUTHORS CONTRIBUTION

Yessy Yusnita was very important in coming up with the idea for the study, developing the methodology, doing the formal analysis, investigating, and writing the first draft.

Nur Hamiza Adenan played a role in coming up with the idea for the research, designing the method, doing the formal analysis, and checking the results.

Angelalia Roza was involved in coming up with the idea for the study and was in charge of reviewing and editing the manuscript.

DECLARATION OF COMPETING INTEREST
The authors declare that there are no conflicts of interest related to this study.

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